K. J. Arrow's Broad Sense on Social Order
—Some Ideas in Social Choice and Individual Values (1951) —
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1. INTRODUCTION

Kenneth Joseph Arrow (1921–) is one of the most renowned American economists of the 20th century. Along with J.R. Hicks, he received a Novel Prize in 1972 as the youngest winner of the honor, for his contributions in the development of welfare economics and a proof of the existence of general equilibrium. This former contribution was published as Arrow's first book, Social Choice and Individual Values (hereafter, SCIV), issued in 1951. The aim of this study is to show that this book covers not only welfare economics but also the study of politics, philosophy, and statistics. From this viewpoint, Arrow displays in his earliest work a multidisciplinary insight into the study of economic and social problems.

The best-known portion of SCIV is the impossibility theorem, which proves that social decision making cannot be led by aggregating the choices of individuals, even under the five simple conditions we presume in a democratic society: (1) unrestricted domain, (2) Pareto efficiency, (3) independence of irrelevant alternatives, (4) sovereignty, and (5) nondictatorship. He sets an axiom of transitivity, and the theorem shows that the rule of transitivity is not satisfied when these conditions are assumed to aggregate individual orderings. This mathematically rigid conclusion drew an immediate response from the academic economics community. Many book reviews were published in economic journals like Econometrica, Economica (New Series), and The Economic Journal. From these book reviews, SCIV was highly evaluated within a stream of welfare economics: as one states, "[i]ts place in the history of welfare economics is assured (Houthakker 1952, 358)."

Indeed, SCIV possesses a theoretical background that aligns with new welfare economics. A social welfare function constructed by A. Bergson denied the comparability of interpersonal utility; it was different from "old" welfare economics represented by A. Pigou (Bergson 1938). Arrow constructs his own social welfare function in SCIV: this was the same tack that Bergson and Samuelson had previously taken. Moreover, the compensation theory of Hicks and Scitovsky is also investigated in this book. These assumptions and discussions are made on the basis of the impossibility theorem laid down in SCIV; therefore,
ties to the theories inherent in the new welfare economics were emphasized.

However, this understanding seems to limit an implication of SCIV to a narrow discipline of economics. When SCIV was published, it had been widely read by practitioners across the social sciences, and reviews were published in several journals outside the domain of economics, like American Sociological Review, American Catholic Sociological Review, and Ethics. The procedure by which to collect decisions had been studied outside of economics, forming a long history of the study of voting systems. Collective decision making as found at the heart of voting systems was first explored by the French philosophers Jean–Charles de Borda and Marquis de Condorcet in the late 19th century; their works offer the case of the paradox of voting, wherein the ordering of each preference ultimately leads to contradictions among them. Arrow’s impossibility theorem has been considered for some time as an exploration of how to escape these paradoxical consequences. As Samuelson (1967) points out with regard to SCIV’s contribution to the creation of mathematical political science, the work has a wide applicability, as well as implications that extend beyond the discipline of economics.

Arrow was sufficiently aware of the wide-ranging spectrum of problems bestowed upon our society by virtue of social decision making; it was more than a welfare economics problem. Such awareness apparently stemmed from his thinking and circumstances at the time he wrote SCIV. In this study, I will examine Arrow’s ideas and thoughts as they appear in SCIV. In the next section, I refer to a broad scope of SCIV and a distance from welfare economics. I then proceed in the third section to examine his notion of relevance between SCIV and philosophical ideas of traditional philosophers. In the fourth section, I refer to Arrow’s thoughts on equality, freedom, and economic calculation controversy, as long as they appear in SCIV. In the fifth section, I provide concluding remarks.

2. THE SCOPE OF SOCIAL CHOICE AND INDIVIDUAL VALUES
2.1 SOCIAL CHOICES: A BROAD MEANING

The wide range of subject matter discussed within SCIV is outlined within its introduction. SCIV starts with this sentence:

In a capitalist democracy there are essentially two methods by which social choices can be made: voting, typically used to make “political” decisions, and the market mechanism, typically used to make “economic” decisions. (Arrow 1951, p. 1)

In SCIV, a voting system is essentially analogous to the market mechanism; in terms of scope, they are treated equally as themes throughout the book. This notion derives from a universal and general expression that does not limit a problem to the domain of economics, and such is
the case with his definition of the social welfare function. The mathematical and logical approach taken throughout SCIV gives it strong applicability to any community in aggregating individual choices (Shackle 1951, p. 430).

The equal positions of voting and the market mechanism derive from the broader sense of social choice. In the next few lines of SCIV, Arrow introduces several types of social choice that differ from each other, depending on the political and social systems of the society at hand. According to Arrow, the types of social decision making are voting, the market mechanism, social decisions by single individuals, and traditional rules. The first two are categorized as a matter of consensus-building, in aggregating individual choices that lead to a social decision among the members of a society; the second two ways are classified as dictatorship and the conventions of single individuals, and custom, respectively, and they lead to social decisions in the absence of the amalgamation of individual choices. Any one of them could be a rational system because, in a sense, they can lead to only one conclusive decision within a society (Arrow 1951, p. 2). According to the thinking in SCIV, Arrow would consider each of dictatorship, convention, and consensus inclusive in facilitating social decision making.

Nonetheless, these types of decision making align with a variety of political and social systems. In the Western liberalism culture, there would have been no agreement that a dictatorship is an acceptable means of making social choices. On the other hand, convention results in a study of comparative cultural analysis or religion. Therefore, the scope of Arrow is focused on Western democratic decision makings (Houthakker 1952, 358). Dictatorship and convention are excluded from an investigation though; they remain methods for social decision. From these reasons, throughout SCIV, differences in political and social systems are important considerations when exploring the issue of social choice. Indeed, there is a discussion in chapter seven of SCIV that touches upon a problem with the social decision process, and it relates to political and social systems.

While the broad applicability and implications of SCIV are certainly considered in the work, the rules and conditions of the theorem are specific to economics, in the way that Arrow considers the social decision-making process. Actually, in SCIV, Arrow’s main references to economics in the introduction are to two points—namely, rationality and welfare judgment—which are formulated to align with the assumptions of the theorem. The transitivity axiom is a formal notation of rationality in economics, and the denial of comparability of interpersonal utility is translated into the condition of Pareto optimality, an economic principle whereby an entity chooses a better state of society among various sets of resource allocation, without calculating the welfare of individual cardinal utility.

Arrow was not interested in where his work “belonged,” precisely. Many book
reviews recognized *SCIV* as a work of new welfare economics; nonetheless, the reaction of some welfare economists to *SCIV* varied. For example, Little (1952) states that “Arrow’s work has no relevance to the traditional theory of welfare economics, which culminates in the Bergson–Samuelson formulation” (p. 425). Bergson (1954) also states that “The theorem has little or no bearing on welfare economics” (p. 243). In the notes to the second version, Arrow refers to these arguments as “elementary confusions or word play,” and states that while

Little, Bergson, and Mishan all agree that my theorem is not part of welfare economics….The most obvious remark to be made is that one can hardly think of a less interesting question about my theorem than whether it falls on one side or another of an arbitrary boundary separating intellectual provinces. Since Little and Bergson (and also Samuelson, according to Little) accept the Bergson social welfare function as part of welfare economics… [it] has something to do with the public adoption of economic policy, and it is hard to see how any study of the formation of social decisions can have “no relevance to” or “no bearing on” welfare economics. (Arrow 1963, p. 108)

Moreover, he wrote in 1963 that “the concept of social choice concerns to both the problem of the content of philosophy, economics, and politics, and mathematical logics and methodological and technical problems of other mathematical disciplines.” In other words, *SCIV* is in its broadest sense a formal consideration of social choice, and economics is but one approach an individual can take.

### 2.2 LIMITATION OF THE IMPOSSIBILITY THEOREM

In *SCIV*, Arrow seems not to embrace the applicability of some conditions considered basic ideas in welfare economics. Most noteworthy are the two economic assumptions noted in the previous section, which are deliberatively declined in *SCIV*. Let us look at each of these, in turn.

(1) Rationality: Arrow seems to be doubtful of an assumption of rationality. For him, the rationality principle is problematic, for three reasons. The first and second reasons relate to a game-theoretic aspect of rationality. The first is that *SCIV* excludes the fun of playing a game.

No consideration is given to the enjoyment of the decision process as a form of play. There is no need to stress the obvious importance of the desire to play and win the game as such in both economic behavior and political. That such considerations are real and should be considered in determining the mechanics of social choice is not to be doubted; but this is beyond the scope of the present study. (Arrow 1951, p. 7)
The second is the particular case of game theory that an individual will avoid divulging a true preference, in order to receive a rather rich profit. Arrow explains thus:

The point here...is that, once a machinery for making social choices from individual tastes is established, individuals will find it profitable, from a rational point of view, to misrepresent their tastes by their actions, either because such misrepresentation is somehow directly profitable or, more usually, because some other individual will be made so much better off by the first individual’s misrepresentation that he could compensate the first individual in such a way that both are better off than if everyone really acted in direct accordance with his taste. (Arrow 1951, p. 7)

The third reason Arrow considers rationality problematic is that there are various concepts of rationality. An economic study generally assumes rational behavior, and the transitivity rule is a key concept that guides this premise within a theory. The transitivity rule also formally represents rationality in SCIV. Arrow seems to believe that rationality, as a transitivity rule, may not always hold:

Many writers have felt that the assumption of rationality, in the sense of one-dimensional ordering of all possible alternatives, is absolutely necessary for economic theorizing....There seems to be no logical necessity for this viewpoint: we could just as well build up our economic theory on other assumptions as to the structure of choice functions of the fact seemed to call for it. (Arrow 1951, p. 21)

Arrow here seems to leave room for the possibility that social choice, or even economics, does not necessarily need transitive consistent ordering.

(2) Pareto efficiency: Arrow thinks that Pareto efficiency is not a sufficient condition for social decision making. Arrow believes that social choice is decided not by optimality, but by desirability.⁸ Pareto efficiency decides only the sets that exhibit the best allocation of resources. To choose a particular set among these sets depends on the making of a value judgment. Furthermore, those sets of Pareto-efficient changes depend on the initial distribution: if an initial supply changes, the options of desirable, Pareto-efficient sets may also change:

General competitive equilibrium above all teaches the extent to which a social allocation of resources can be achieved by independent private decisions coordinated through the market. We are assured indeed that not only can an allocation be achieved, but the result will be Pareto efficient. But, as has been stressed, there is nothing in the process which guarantees that the distribution be
just….Thus even under the assumptions most favorable to decentralization on decision making, there is an irreducible need for a social or collective choice on distribution. (Arrow 1972, p. 128)

He distances himself, here, from the standpoint of welfare economics. In the following sentences in the second edition of SCIV (1963), he comments on the difference between welfare economics and social choice:

A more natural division between welfare economics and the theory of social choice, if there is any point in locating one, is provided by defining the former as the implications of Pareto optimality. I take this to be Mishan's meaning in reading my work out of the honorific domain of welfare economics. In the terms used in this book, instead of seeking a social ordering, we confine ourselves to the unanimity quasi–ordering. (Arrow 1951, p. 108)

This implies that Arrow, within SCIV, does not believe in the assumed, essential need for Pareto efficiency.

Through his impossibility theorem, Arrow appears to criticize the consistency of social decision making in a democratic society. However, Arrow believes there remains the possibility of the existence of a reasonable way of making social decisions, beyond the scope of the impossibility theorem. The impossibility theorem does not imply that it is absolutely impossible to facilitate social decision making. He writes:

If we wish to make social welfare judgments which depend on the values of all individuals, i.e., which are not imposed or dictatorial, then we must relax some of the conditions imposed. The only condition that remains to be eliminated is the one stating that the method of forming a social ordering should work properly for a wide range of sets of individual orderings. (Arrow 1951, p. 60)

Arrow believes that there is room to mitigate some of the conditions he had formulated as parts of the impossibility theorem. Many studies have further developed Arrow's theorem; in one sense, numerous works issued in the wake of SCIV follow this way out, which Arrow had suggested. Baumol (1952) correctly foresees that

This result is less disastrous for welfare theory than might first appear. Clearly, lower acceptability standards are required. Fortunately, rather little sacrifice may be involved…Similarly, occasional care-fully screened departures from transitivity may not be too serious. Thus the book need not be viewed as cause for alarm. Rather, it is an illuminating and timely discussion of the problems involved in setting up standards and procedures for group decisions based somehow on


individual preferences. (p. 110)

The result of Arrow’s impossibility theorem requires further theoretical research, if we are to study acceptable conditions that are less strong than Arrow presumes.

Returning to SCIV, what Arrow actually expects in SCIV is the possible mitigation of Condition 1, which consists of a part of the impossibility theorem:

The only condition that remains to be eliminated is the one stating that the methods of forming a social ordering should work properly for a wide range of sets of individual orderings. (Arrow 1951, p. 60)

Arrow considers, in the remainder of SCIV, a possible reconstruction by which to reach a desirable social choice that is acceptable to the members of society.

3 RESEMBLANCE BETWEEN A STATE OF SOCIAL WELFARE FUNCTION AND POLITICAL PHILOSPHY

3.1 SIMILARITY OF SOCIAL ATTITUDES AS A BASIS OF SOCIAL CONSENSUS

Technically speaking, there are two ways of fulfilling the mild conditions of the impossibility theorem. First, he tests a slight change in Condition 1—the unrestricted domain—which states that social orderings should work for a wide range of sets of individual orderings. Second, in the event of an unsuccessful first modification of conditions, he considers similarities among social attitudes. In a course of this modification, the paradox occurs even after a modified condition is introduced. Even if Condition 1 were modified, there remains no knowledge about the method by which each individual orders social alternatives. Therefore Arrow proceeds to the next step; he believed that the positive valuation of similar social attitudes is essential to deriving a desirable social consensus.

Similarity among social attitudes is an essential factor to building social choice: in his function, social attitudes accompany individual values. From his proof of the impossibility theorem, he refers to the state that such a similarity among social attitudes is to share. “[I]t must be demanded there be some sort of consensus on the ends of society, or no social welfare function can be formed” (p. 83). He states:

The assumption of Section 1, that of complete agreement among individuals on the ordering of social alternatives...is at the basis of a great portion of political philosophy, namely, the idealist school...This view is expressed in the works of Rousseau, Kant, and T. H. Green, among many others. (Arrow 1951, p. 81)

Arrow here compares Arrow's function to the idealist philosophy. Kant introduces
three different principles vis-à-vis behavior: (1) technical imperatives, (2) pragmatic imperatives, and (3) moral imperatives. For Arrow, pragmatic and moral imperatives are especially important concepts here: pragmatic imperatives are used to pursue individual happiness, while moral imperatives drive behavior related to the notion of duty. In relation to the social welfare function, pragmatic imperatives correspond to individual ordering, while moral imperatives correspond to social ordering. If we were to guess that moral motives help build similarity among the social attitudes of the people, a desirable social welfare function can be led by a state where moral imperatives give all individuals a particular tendency to undertake individual ordering over a society—one that is somehow compatible with pragmatic imperatives.

3.2 RELEVANCE BETWEEN A SOCIAL STATE AND POLITICAL SYSTEM

How can we accomplish such a state in a democratic society? Arrow refers to the work of Rousseau:

It is from this point of view that Rousseau discussed the relative merits of different forms of government. In this aspect, the case for democracy rests on the argument that free discussion and expression of opinion are the most suitable techniques of arriving at the moral imperative implicitly common to all. (Arrow 1951, p. 85)

For Rousseau, the general will is a state in which all the members of a community agree with the rules of society and have consistent opinions on them. According to Rousseau, it is important that people provide opinions of a general will through free discussions: this activity of interchange among individuals accomplishes general will. Thus, Arrow compares his social welfare function with the philosophical literature on social consensus, within the context of a democratic political system.

Arrow’s social welfare function enables to compare his idea with the idealist tradition. It can be seen in his original interpretation of the social welfare function. In his formulation of the social welfare function, he emphasizes that social choice orderings are constructed not by individual preference, but by individual values (Arrow 1951, p. 4, 72). It is in contrast to Bergson’s social welfare function since social orderings of Bergson’s function derive directly from the constitution of individual orderings based on individual preferences. However, in the case of Arrow’s function, individuals decide their orderings on the basis of their evaluations through relationships with other individuals. Therefore, Arrow’s social welfare function considers the cognizance of others’ existence.

The difference between Bergson and Arrow in terms of their understanding of the social welfare function highlights Arrow’s idea of how an individual connects with society.
An individual is not an isolated existence; each one has ties with others, and social orderings flexibly change through activities that mutually affect each individual. Arrow clearly says that “the values held by any one individual will vary with variations in the tastes of all” (1951, p. 71). Arrow thinks of dynamic process of society in which individuals interacts. It mirrors Rousseau’s democratic process through free discussion to reaching a consensus. That is how Arrow expresses philosophical implication of his social welfare function.

Importantly, Arrow does not intend to treat these factors in a traditionally philosophical manner. Arrow insists that techniques that make use of statistics can possibly facilitate the comparison of such opinions and lead general will:

This model has much in common with the statistical problem of pooling the opinions of a group of experts to arrive at a best judgment. (Arrow 1951, p. 85)

Arrow here supposes that such a statistical method objectively revaluates individual orderings and gives a better criterion comparable among opinions (Arrow 1951, p. 85). Another is found in a different lines but the same context:

The social ordering of social decisions should be based on the social ordering of social ends plus the use of scientific and statistical methods to limit the amount of ignorance in passing from decisions to end and to limit the effects of the remaining ignorance. (1951, pp. 87–88)

Here too, he suggests the utility of statistical methods in leading the social welfare function. Statistics limit the numbers in a set of alternative preferences to those that truly relate to an ultimate end; it also restrains the uncertainty that can result from ignorance by leveraging probabilistic calculations in a scientific manner. Arrow is interested in the utility of using a statistical method, as doing so would help calculation of a desirable social decision.\(^\text{11}\)

4 FURTHER CONSIDERATIONS OF SOCIAL ATTITUDES: ARROW’S EXAMPLE

4.1 EQUALITY

What kind of similarity among individuals vis-à-vis social attitudes does Arrow consider essential? The remainder of this chapter discusses Arrow’s thoughts on several values that are consistent with the social attitudes that tie individuals each other. He provides an example in the following:

Some values which might give rise to such similarity of social attitudes are the desires for freedom, for national power, and for equality. (Arrow 1951, p. 74)
First, equality appears in *SCIV* as a problem of distributional ethics (Arrow 1951, p. 71). According to Arrow, there are two different means of distribution: (1) Pareto efficient and (2) egalitarian. Which to choose of the two depends on the value judgments of an individual. He here considers egalitarian distribution ethical (Arrow 1951, p. 71).

In Arrow’s understanding, the concept of “equality,” from an idealistic viewpoint, relates to both moral and pragmatic imperatives. Equality in terms of moral imperatives is accomplished under the reasoning that equality should be ethically pursued. Equality accomplished through a process involving a pragmatic imperative, on the other hand, has been seen in some work in new welfare economics (e.g., Bergson’s “propositions of relative shares” or Hicks and Scitovsky’s compensation principle). He offers no explicit ideas here, but he seems doubtful of the existence of ethical methods by which to obtain equality. He says:

> Ethical absolutism is unsatisfying to a mind brought up in the liberal heritage, however much specific shortcomings in the liberal formulation are rejected. (Arrow 1951, pp. 84–85)

If we regard equality as being driven solely by moral imperatives, then it is unacceptable for individuals who live in a liberalist society. On the other hand, Arrow has problems with the egalitarian process inherent in the market mechanism, when we could consider equality a matter of pragmatic imperatives.

Any view which depends on consensus as the basis for social action certainly implies that the market mechanism cannot be taken as the social welfare function since that mechanism cannot take account of the *altruistic* motives which must be present to secure that consensus. (Arrow 1951, p. 86; italics added for emphasis)

Eventually, Arrow states that:

> This does not deny the possibility of a limited use of the market as an instrument for achieving certain parts of the social optimum, especially if it is deliberately manipulated to make behavior under pragmatic imperatives coincide with that which would exist under moral ones. (Arrow 1951, p. 86)

He deliberately mentions his idea concerning the concept of equality. However, given the explanation above, it seems that he expects there to be room for an egalitarian method, by way of deliberate market manipulation.

4.2 OTHER EXAMPLES: THE DECISION-MAKING PROCESS AS A VALUE
Second, other examples of social attitudes for which there are similarities among individuals are freedom and national power. These are touched upon in an investigation by Arrow, wherein he considers the decision-making process itself a value. In the previous discussion, values are denoted as external factors by which an individual brings about social orderings; nonetheless, there is a further interpretation of the social welfare function, which allows us to understand that values are internal factors that lead to social orderings:

One especially interesting analysis...considers that, among the variables which taken together define the social state, one is the very process by which the society makes its choice. This is especially important if the mechanism of choice itself has a value to the individuals in the society. (Arrow 1951, p. 89)

Technically, this process is led by an interpretation of social welfare function where \( x_n \) is a decision-making process to decide social ordering, while considering all the previous factors, \( x_1, \ldots, x_{n-1} \), when there are alternatives, \( x_1, \ldots, x_n \) in a set \( S \). An \( n \)-th decision is made by a set \( (x_1, x_2, \ldots, x_{n-1}) \). Similarly, an \( n + 1 \)-th decision will be considered by a set \( (x_1, x_2, \ldots, x_n) \). If we set the last vector to always and continuously consider the other vectors as part of the decision-making process, the last vector will play the role of individual preference toward the decision-making process itself. For instance, if an individual prefers majority-voting to unanimous-voting once, he or she will consider the fact that he or she preferred majority-voting in the previous stage, when deciding upon ordering in the present stage. What this interpretation of the social welfare function means is that preferences vis-à-vis freedom or national power—or even equality—can be internalized as individual values for which they have a preference in the social welfare function. The concepts of freedom and national power can be put into the social welfare function as vectors of an individual's socio-psychological tendency toward a similarity of social attitudes. Especially, Arrow mentions an example of a choice within the decision-making process itself, between a liberalist economy and a socialist economy; he insists upon its importance thus:

For example, an individual may have a positive preference for achieving a given distribution through the free market mechanism over achieving the same distribution through rationing by the government. If the decision process is interpreted broadly to include the whole socio-psychological climate in which social decisions are made, the reality and importance of such preferences, as opposed to preferences about the distributions of good, are obvious. (Arrow 1951, p. 90)

In this case, it happens that a particular means of distribution may not be chosen,
because the decision-making process is deemed unacceptable. He explains:

For example, the belief in democracy may be so strong that any decision on the
distribution of goods arrived at in other ways, even though all individuals might
have preferred the second distribution of goods to the first of it had been arrived at
democratically. Similarly, the desire for a dictatorship or for a particular dictator
may be overwhelming under certain conditions. (Arrow 1951, p. 90)

This process also hinges on Arrow’s characteristic understanding of the social
welfare function, because the process he suggests here is able to understand by his own
interpretation of social welfare function.

4.3 CONTROVERSY SURROUNDING ARROW’S DELIBERATE ATTITUDE TOWARD
ECONOMIC CALCULATION

The discussion above of Arrow was originally based on a series of controversies on
the possibility of economic calculation in a socialist economy. Since the 1930s, a series of
discussions on an economic calculation problem were deemed controversial. In his
precedent work in the 1920s, Ludwig von Mises insists that there is no method of operating
an economic system efficiently within socialist economies (Mises 1920). What von Mises
strongly asserts is the absence of a capital price throughout the production process, given
that within a socialist economy, the government holds the means of production. Von
Mises’s criticism goes further by stating that a planned economy is an inefficiently
functioning system that lacks pricing. In contrast, Oskar Lange speaks of the feasibility
of the central government of a planned economy calculating a production cost (Lange
1936-1937). Lange’s insistence is based on his assertion that a government can assign a
calculated price to each product, depending on careful regulation of excess supply and
demand informed by industrial sections in the latest production period. Lange thought that
a calculated cost would reach an optimum through a trial-and-error process.

According to Lange, price can be calculated when there is the freedom to choose
both consumption and occupation, for he presumes that an individual demands efficiently.
On the other hand, Dobb was critical of a reliable individual’s ability to choose. Because of
this, one of Dobb’s criticisms was with regard to consumer sovereignty. Arrow’s comment
on Dobb focuses on Dobb’s insufficient criticism of consumer sovereignty. Arrow suggests
that Dobb means that there is another way by which a planned economy can function,
which is neither the market mechanism nor Lange’s market socialism—although, Arrow
complains, “he does not specify them” (1951, p. 84). On this matter, Arrow says that:

Dobb’s work presents clearly the dilemma posed by accepting the doctrine of
consensus as a foundation of social ethics. Empirically, we can reject the idea that the consensus can be found in the expressed individual wills. If consensus is to be found in the moral imperative, what is the basis for it? (1951, p. 84)

Arrow denies treating ethics as a set of pragmatic imperatives represented by individual wills: on the other hand, in recalling the discussion of Arrow’s thoughts on equality, one realizes that Arrow is also reluctant to treat ethics as moral imperatives. How, then, does Arrow evaluate Lange’s market socialism? He does seem to provide clues in his discussion of centralized and decentralized planning:

It may well be argued that centralized planning will necessarily reduce this uncertainty since more facts are available to the central planners. (Arrow 1951, p. 88)

Central planners leverage fully the data they can use. Arrow allows us to see that uncertainty can be reduced in a centralized economy whenever there are errors stemming from insufficient data; he is optimistic in saying that errors can be mitigated “if a rational method of planning against uncertainty is adopted” (Arrow 1951, p. 88). And this is an advantage inherent in a centralized economy that Lange expects a market socialism economy to bear this characteristic.

Arrow provides no explicit, conclusive remarks on a series of debates on the economic calculation controversy; nonetheless, from the results of the preceding two sections, it seems he is inclined to subscribe to Lange’s viewpoint. His impossibility theorem does not rest on the feasibility of leading social decision making in the absence of a similarity of social attitudes among the populace. If there were no possibility of constructing the social welfare function by making use of pragmatic imperatives, it would at least be natural to believe that he considers a method involving market socialism to be a possible compromise between moral imperatives and pragmatic imperatives, even if a concrete method is not clear. Returning to the previous section on Arrow’s thoughts on equality, one sees that he speaks of a deliberate manipulation of the market. Indeed, Arrow was not averse to Lange’s thinking—at least not in the late 1940s, when he wrote *SCIV.*

5 CONCLUSIVE REMARKS

This paper shows that *SCIV*, published in 1951, has important implications not only for welfare economics, but also on a broader scale. As was shown in this study, decision making is a subject that touches many situations that we face in society. He examines a problem of social choice by mathematical way. Arrow formulates social welfare function and insists that techniques of statistics possibly compare individual orderings and help to
lead the consensus. From this view point, in *SCIV*, Arrow appears to be a mathematician. On the other hand, Arrow was sufficiently aware of the depth of the problem of social choice, and he scrutinized it in *SCIV* especially, that scrutiny appears with regard to interpretations of the social choice function. The moral imperatives of the idealist tradition are presented as a possible bypass for reaching similarity of social attitudes, even though he does not rest on a solution by morality. In this sense, Arrow's investigations leave room for the addition of political, philosophical, and statistical interpretations. Arrow seems to proportion formalization with philosophical explanation of social welfare function. While his viewpoint was further elucidated by way of the supplemental note in the second edition, it is nonetheless consistent in the first edition as well.

Arrow may appear to be an economic theorist, given that it was for his work in this discipline that he received a Novel Prize. However, an examination of his full corpus makes it clear that he created a considerable number of nontheoretical works in a variety of areas, including uncertainty, organization, and health care. In these works, Arrow does not always take for granted the assumptions of modern economics; in my opinion, his ability to transcend these assumptions derives from his ability to see reality. He believes in the effectiveness of the market mechanism on one hand, but on the other, he realizes the limitations of the applicability of the market mechanism, which is seen in this paper. In this regard, images of Arrow as solely a theory-minded researcher need to be amended. 14

This view of Arrow as a multidiscipline researcher seems to be consistent with the contents of *SCIV*, where Arrow appears as a philosopher and a mathematician who attempted to solve social problems through the use of 20th-century mathematical science.

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NOTES

1 *Social Choice and Individual Values* has three versions (1951, 1963, 2012). The content remains largely unrevised, except that Arrow added a supplemental chapter to the 1963 version. That supplemental chapter comprises a summary of 10 years of development of social choice theory, and several comments by Arrow. The third version features a forward by Eric S. Maskin.

2 I followed the ordering of descriptions as found in the original literature. Some explanations of Arrow’s impossibility theorem bypass the condition of sovereignty. For example, a foreword written by Eric S. Maskin in *Social Choice and Individual Values* explains five conditions, while omitting the condition of sovereignty and including the axiom of transitivity.

3 Others are *The Journal of Business of the University of Chicago* and *Southern Economic Journal*.

4 The rest is *The Southwestern Social Science Quarterly*.

5 The classification by Arrow follows the work of Knight (1947). Knight classifies social choice in terms of dictatorship, convention, and consensus. As Arrow explains, he
distinguishes consensus in terms of voting and the market mechanism.

6 In line with such concerns, there has been a debate about what SCIV is expected to understand in the context of capitalist democratic society (Amadae 2002).

7 This is written in the preface for the Japanese-language translation of the second version, which was published in 1963; this preface is not found in the English-language version.

8 Baumol’s comment on SCIV captures Arrow’s belief in this point. Baumol states: “This should not be confused with a problem familiar to welfare theory, that of finding an optimum social choice (welfare) function. Thus, if there are any acceptable social choice functions, the optimum function must presumably be among them.” (Baumol 1952, p. 110)

9 Condition 1 is defined as there being individual orderings among a wide range of possible sets of alternatives in a society, \( R_1, \ldots, R_n \), compatible with individual orderings among three alternatives in a particular set \( S, T_1, \ldots, T_n \). If \( x \) is preferred to \( y \) in set \( S \), then \( x \) is preferred to \( y \) in all alternatives when \( x \) and \( y \) are in set \( S \). Written formally, if \( x \ R y \), then \( x \ R y \). A modification is done by reforming individual orderings, \( T_1, \ldots, T_n \), to individual orderings, \( Q_1, \ldots, Q_n \). \( Q \) suggests that not only is \( x \) preferred to \( y \), but also that \( x \) is preferred and indifferent to \( y \).

10 He here recognizes the modified condition as a formal representation of the new welfare economic presumption. Arrow points out that there are two characteristics in the conditions of welfare economics. One is the fact that welfare economics considers only an aspect of economic welfare. Vectors for economic resources like consumption, leisure, and saving are taken into account when two different social alternatives are compared for the sake of making a desirable social choice. Another is that welfare economics postulates Pareto efficiency. He forms the individualistic assumption, as it matches what new welfare economics postulates. The quotations above demonstrate the unsatisfactory consequences of the conditions of new welfare economics, to dismiss the paradox of choosing social states. He concludes that “the results…suggest strongly that the difficulties in forming social welfare function arise from the differing social attitudes which follow the individualistic hypothesis.” (Arrow 1951, p. 69)

11 Arrow’s recognition of mathematics and statistics is natural, in a sense. When we examine his pre-SCIV study career, we find that he was a rather mathematical and statistics-minded man, with a keen interest in his early days in those two disciplines and philosophy. After experiencing the Great Depression of 1929 in his youth, Arrow majored in mathematics at the City College of New York and received a Bachelor of Arts in mathematics in 1939. According to his retrospective, he eagerly read books in mathematics, statistics, and philosophy there and continued to study statistics at Columbia University. The subject of his masters-level thesis was the stochastic process.

12 A debate on economic calculation problem is firstly provoked by Dickinson in the 1930s. Lange’s discussion is based on the debate by Dickinson. Dickinson insists that distributive equilibrium in Socialist economy is possible by mathematical calculation of equations to lead a price. Possibility of leading a price by mathematical calculation is firstly referred by Barone (Barone 1938).

13 Dobb’s objection against consumer sovereignty are seen in several literatures by Dobb (Dobb 1955, 1969). For a detailed discussion, see also, Amadae(2002), pp. 90-96.

14 In the bibliographical literature, Feiwel speaks of Arrow’s multifacetedness, pointing out that he is a musician who composes music that features a variety of patterns. This point is similar to that made in the conclusion of this study—namely, that Arrow undertook research in many areas of economics—but these conclusions also differ, in that I assert that Arrow’s thinking goes well beyond the discipline of economics.
REFERENCES


